# UNCLASSIFIED

# UNCLASSIFIED

# PRINCETON UNIVERSITY

## DEPARTMENT OF AERONAUTICAL ENGINEERING

61-4-5
XEROX

SOME BASIC CONSIDERATIONS REGARDING
THE LONGITUDINAL DYNAMICS OF AIRCRAFT
AND HELICOPTERS

by

Howard C. Curtiss, Jr.

Department of Aeronautical Engineering
Princeton University

Report No. 562                    July 1961

Approved by:

*A. A. Nikolsky*

A. A. Nikolsky
Project Leader

## FOREWORD

The research in this report was conducted by the Department of Aeronautical Engineering of Princeton University under the sponsorship of the United States Army Transportation Research Command, as Phase 1 of work under the ALART Program.

The work was performed under the supervision of Professor A. A. Nikolsky, Department of Aeronautical Engineering, Princeton University.

This work was administered for the United States Army by Mr. John Yeates.

## TABLE OF CONTENTS

SUMMARY

A discussion of the longitudinal motion of an airframe is presented. General relationships between the stability derivatives of the airplane and the single rotor helicopter are considered. It is shown that the basic character of the longitudinal motion is primarily determined by the angle of attack stability and the velocity stability. The variation in the modes of motion produced by these two stability derivatives is presented.

Consideration of the relationships between the flight variables in the modes of motion is included.

# SYMBOLS

$g$ — Acceleration due to gravity.

$I$ — Moment of Inertia of aircraft about center of gravity.

$M$ — Pitching moment about the center of gravity, positive nose up.

$M_\alpha, M_{\dot\theta}, M_u$ — Rate of change of pitching moment with variable indicated in subscript, the others held constant.

$m$ — Pitching moment divided by moment of inertia $\left(\frac{M}{I}\right)$; mass of aircraft. ___

$t$ — Time

$u$ — Flight velocity

$W$ — Weight of aircraft

$X$ — Horizontal force along an axis fixed to the aircraft, initially aligned with the wind, positive forward.

$X_\alpha, X_u$ — Rate of change of horizontal force with variable indicated in subscript, the others held constant.

$x$ — Horizontal force divided by the mass of the aircraft. $\left(\frac{X}{m}\right)$

$Z$ — Vertical force, perpendicular to $X$, positive downward.

$Z_\alpha, Z_u$ — The rate of change of vertical force with the variable indicated in the subscript, the others held constant.

$z$ — Vertical force divided by the product of the mass of the aircraft and the trim velocity. $\left(\frac{Z}{m u_o}\right)$

$\alpha$ — Aircraft angle of attack, positive nose up.

$\lambda$ — Root of characteristic equation.

$\theta$ — Aircraft pitch angle, positive nose up.

$\sigma$ — Real part of root of characteristic equation (damping of oscillation).

$\omega$ — Imaginary part of root of characteristic equation (frequency of oscillation).

$\delta$ — Control deflection.

2

Subscripts and superscripts

$( \ )_0$      initial or trim value.

$\Delta( \ )$      perturbation from initial value.

$( \ )_I$      a quantity representing the magnitude and phase of a variable.

$\dot{( \ )}$      derivative with respect to time.

3

# INTRODUCTION

The general nature of the transient longitudinal motion of conventional subsonic airplanes has been well known for a number of years. The classical motion consists of two oscillatory modes, one proceeding at approximately constant velocity (referred to as the short period), and the other proceeding at approximately constant angle of attack (referred to as the phugoid). However, the restrictions on the aerodynamic stability derivatives for the occurrence of this type of motion do not appear to be well known. Many other flying machines, the single rotor helicopter, for example, exhibit dynamics considerably different from this classical pattern. Since the longitudinal motion of both of these types of aircraft, and other flying machines, can be described by essentially the same equations of motion, it would be expected that there would exist a number of basic similarities in the transient motion.

It is the purpose of this report to provide a general viewpoint for the longitudinal dynamics of an aircraft by indicating and utilizing the fundamental similarities that usually exist. The basic restrictions on the aerodynamic stability derivatives necessary for the occurrence of classical longitudinal motion are indicated. Root locus techniques (Reference 1 & 9) are used to present, in a general way, the influence of the static stability derivatives. Only the terms which contribute to the essential features of the dynamics will be considered. Thus some terms usually included in stability analysis are neglected, since they contribute small differences to the motion.

4

Also discussed is the variable content of each mode, which is another property of the equations of motion. This characteristic of the equations makes it possible to obtain a good physical picture of each mode, and to determine the variables and the stability derivatives that are of importance in each mode.

The typical longitudinal modes of the single rotor helicopter are discussed. It will be seen that many types of rotor lifted craft fall into this category, and that certain classes of dynamics are more or less inevitable.

# DISCUSSION

The conventional linearized rigid-body equations which describe the perturbed longitudinal airframe motions about a straight and level flight path, may be written in the following form:

$$X_u \Delta u - m \Delta \dot{u} + X_\alpha \Delta \alpha + X_{\dot\alpha} \Delta \dot\alpha + X_{\dot\theta} \Delta \dot\theta - W \Delta \theta = X_\delta \delta$$

$$Z_u \Delta u + Z_\alpha \Delta \alpha + (Z_{\dot\alpha} - m u_o) \Delta \dot\alpha + (m u_o + Z_{\dot\theta}) \Delta \dot\theta = Z_\delta \delta$$

$$M_u \Delta u + M_\alpha \Delta \alpha + M_{\dot\alpha} \Delta \dot\alpha + M_{\dot\theta} \Delta \dot\theta - I \Delta \ddot\theta = M_\delta \delta \qquad (1)$$

The coordinate system is initially aligned with the relative wind and is fixed to the body. The $X$ axis is taken positive forward and the $Z$ axis positive downward. The control response will not be considered. Only the character of the homogeneous equations is investigated.

In order to describe the longitudinal motion of the helicopter with these equations, it is necessary to make additional assumptions to the assumptions implicit in this form of the equations.

1. Coupling effects between the longitudinal and lateral motions are assumed to be negligible. The helicopter differs in this respect from the airplane due to the fact that the rotor is not symmetric, and thus aerodynamic coupling is present. The lateral motion of the helicopter thus produced is assumed to have a small effect on the longitudinal motion.

2.  The rotor reacts instantaneously to changes in flight varia-
ables, i.e., its perturbed position in space can be described by the
instantaneous values of $\Delta\alpha$ , $\Delta u$ , and $\Delta\dot{\theta}$ .  This assumption
is valid for investigations of the transient dynamics of the heli-
copter, since the natural frequency of the blade flapping motion is
much higher than the natural frequency of the fuselage motion.

3.  The rotational speed of the rotor is constant.  It is assumed
that rotor speed variations will not influence the basic character of
the motion, which is of primary concern here.

A number of terms included in the above equations are, in general,
not important, and will be neglected in this analysis.  These are
$X_{\dot{\theta}}\,\Delta\dot{\theta}$ , $Z_{\dot{\theta}}\,\Delta\dot{\theta}$ , $X_{\dot{\alpha}}\,\Delta\dot{\alpha}$ and $Z_{\dot{\alpha}}\,\Delta\dot{\alpha}$ .  Also, the term in the
moment equation $M_{\dot{\alpha}}\,\Delta\ddot{\alpha}$ will be neglected.  The derivative $M_{\dot{\alpha}}$ con-
tributes primarily to the damping of the short period motion and does
not influence the basic character of the motion.  Therefore, consider-
ation of the dynamics will be restricted to the following equations:

$$X_u\,\Delta u - m\Delta\dot{u} + X_\alpha\,\Delta\alpha - W\Delta\theta = 0$$

$$Z_u\,\Delta u + Z_\alpha\,\Delta\alpha - mu_o\,\Delta\dot{\alpha} + mu_o\,\Delta\dot{\theta} = 0$$

$$M_u\,\Delta u + M_\alpha\,\Delta\alpha + M_{\dot{\theta}}\,\Delta\dot{\theta} - I\,\Delta\ddot{\theta} = 0 \qquad (2)$$

For the following discussion it is convenient to divide each equation
by its inertia term.  Thus equations (2) are to be written as:

$$X_u\Delta u - \Delta\dot{u} + X_\alpha\,\Delta\alpha - g\,\Delta\theta = 0$$

$$Z_u\,\Delta u + Z_\alpha\,\Delta\alpha - \Delta\dot{\alpha} + \Delta\dot{\theta} = 0$$

$$m_u\,\Delta u + m_\alpha\,\Delta\alpha + m_{\dot{\theta}}\,\Delta\dot{\theta} - \Delta\ddot{\theta} = 0 \qquad (2a)$$

7

Since these differential equations are linear with constant co-
efficients, the solution will be of the form $\Delta x(t) = \alpha_1 e^{\lambda t}$,
$\Delta\Theta = \Theta_1 e^{\lambda t}$, $\Delta u = u_1 e^{\lambda t}$. Substituting these expressions into
the differential equations, canceling out $e^{\lambda t}$, a set of algebraic
equations in three unknowns $\alpha_1$, $\Theta_1$, $u_1$ and the parameter $\lambda$ is
obtained:

$$(x_u - \lambda)u_1 + x_\alpha \alpha_1 - g\Theta_1 = 0$$

$$z_u u_1 + (z_\alpha - \lambda)\alpha_1 + \lambda\Theta_1 = 0$$

$$m_u u_1 + m_\alpha \alpha_1 + (m_{\dot{\Theta}} - \lambda)\lambda\Theta_1 = 0 \qquad (3)$$

It is a property of these equations that there can be non-zero
values of $\alpha_1$, $\Theta_1$ and $u_1$ if, and only if, the determinant of the
coefficients of these quantities equals zero, i.e.:

$$\begin{vmatrix} (x_u - \lambda) & x_\alpha & -g \\ z_u & (z_\alpha - \lambda) & \lambda \\ m_u & m_\alpha & (m_{\dot{\Theta}} - \lambda)\lambda \end{vmatrix} = 0 \qquad (4)$$

Expansion of this determinant results in a fourth order equation in $\lambda$,
the roots of which are called characteristic values or modes of the
system. The individual values of $\lambda$, i.e., the modes indicate the
nature of the transient motion, e.g., a complex pair of values of $\lambda$
represents an oscillation.

Now to each value of $\lambda$ (each complex pair in the case of com-
plex roots) there corresponds a relationship between $\alpha_1$, $\Theta_1$ and
$u_1$. However, it is possible only to solve for ratios of these

8

quantities, since equations (3) are homogeneous. Thus the ratios $\frac{\alpha_1}{\Theta_1}$

and $\frac{u_1}{\Theta_1}$ (referred to here as mode ratios) can be determined for each

$\lambda$ and do not depend upon the input. (The absolute magnitudes, i.e.,

the $\Theta_1$'s depend upon the input). $\frac{\alpha_1}{\Theta_1}$ and $\frac{u_1}{\Theta_1}$ which, in general,

are complex numbers indicate the magnitude and phase relationship be-

tween the independent variables in each mode. As a result, a good

physical picture of the mode and an estimate of the important terms

in the equations of motion with respect to each mode can be obtained.

This analysis will indicate the general nature of $\lambda$ and the

corresponding variable relationships as a function of the stability

derivatives.

In the following discussion it is essential to note the distinction

between the static stability of the aircraft, and the static stability

derivatives.

The static stability of the aircraft can be defined as the change

in pitching moment due to a change in velocity, under the condition

that the vertical force is maintained equal to the weight. The airplane

is statically stable if an increase in speed produces a nose up moment.

From equations (2a) then, an expression for the static stability of the

aircraft can be determined:

$$m_u \Delta u + m_\alpha \Delta \alpha = \Delta m$$

where

$$z_u \Delta u + z_\alpha \Delta \alpha = 0$$

thus

$$\frac{\Delta m}{\Delta u} = m_u - m_\alpha \frac{z_u}{z_\alpha} = \frac{1}{z_\alpha}(m_u z_\alpha - m_\alpha z_u)$$

9

The term $(m_u z_\alpha - m_\alpha z_u)$ is the coefficient of $\lambda^0$ in the characteristic equation. For the sign convention used here, the airplane is statically stable if $m_\alpha z_u - m_u z_\alpha > 0$. For the helicopter in hovering, this reduces to $m_u > 0$.

Thus the static stability of the aircraft depends on both the static stability derivatives, $m_\alpha$ and $m_u$, as well as the force derivatives $z_\alpha$ and $z_u$.

In the following, $m_\alpha$ will be referred to as angle of attack stability and $m_u$ as velocity stability, whereas the static stability of the aircraft is $\frac{dm}{du}$, the total derivative of pitching moment with respect to velocity, with the vertical force maintained equal to the weight.

Before proceeding further, various 2 x 2 minors in the 3 x 3 determinant (4) will be identified. These minors represent limiting cases of the three degree of freedom dynamics, involving only two degrees of freedom, and may or may not represent a good approximation to a mode of the three degree of freedom dynamics. The discussion following will indicate when these approximations are valid. Three minors are identified, one closely associated with the helicopter, the others with the airplane.

1. Hovering minor.

$$
\begin{vmatrix}
x_u - \lambda & -g \\
m_u & (m_{\dot\theta} - \lambda)\lambda
\end{vmatrix} = 0
\tag{5}
$$

This minor arises from the assumption that perturbations in angle of attack (vertical velocity in hovering) do not have significant influence in the horizontal force and moment equations. The term "hovering"

10

is used to identify this minor, since it describes the dynamics of the helicopter near hovering flight, but will, of course, describe the dynamics of any aircraft in forward flight that obeys these assumptions. The locus of roots to this minor depend primarily on the magnitude of the velocity stability ($m_u$) since the relative values of $x_u$ and the damping in pitch ($m_{\dot\theta}$) will be similar for most aircraft. Physically, it would be expected that if the equations are uncoupled ($m_u = 0$) the dynamics would consist of a rapid convergence in pitching rate ($m_{\dot\theta}$), and a slow convergence in velocity ($x_u$) and therefore $|m_{\dot\theta}| \gg |x_u|$. A degree of freedom is considered uncoupled here in the sense that the root associated with the uncoupled degree of freedom can be determined from one equation. The term $-g \Delta\theta$ is from this viewpoint, a forcing function in the horizontal force equation when $m_u = 0$, and, of course, influences the control response but does not effect the roots of the characteristic equation. Only velocity stability (an increase in velocity produces a nose up moment) will be considered. This is typical of the single rotor helicopter. Thus the locus of roots of this minor is obtained from the characteristic equation

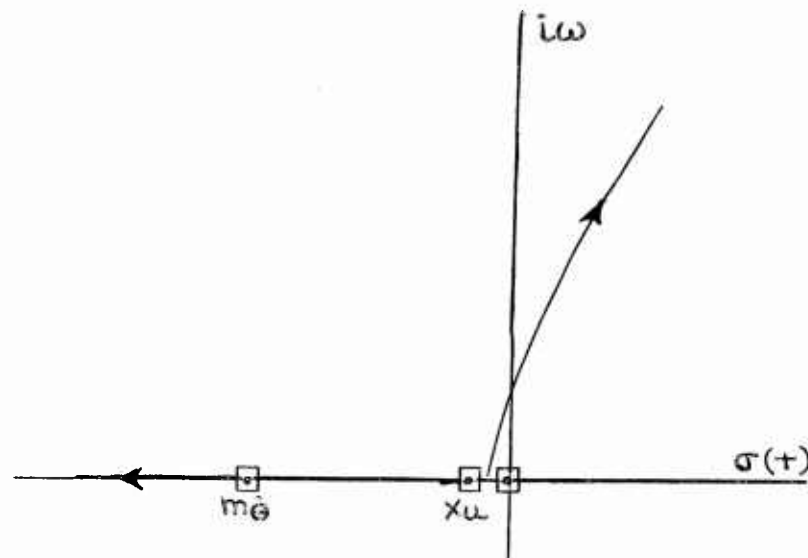$$\lambda \, (m_{\dot\theta} - \lambda)(x_u - \lambda) + m_u \, g = 0$$

Figure 1:   Hovering Dynamics:   Locus of roots for increasing
            Velocity Stability.

From Figure 1 it can be seen that the velocity stability produces an
oscillation which becomes shorter in period and unstable as the ve-
locity stability is increased.   In general, the region of stability
is very small and any appreciable value of $m_u$ will cause the oscil-
latory instability typical of the hovering helicopter.   A description
of this oscillation from a physical viewpoint can be found in Reference
(2).   The magnitude of the $m_u$ above which the motion is unstable can
be determined from Routh's Discriminant.   It is $m_u = -x_u \, m_{\dot\theta}^2$.   These
dynamics will be characteristic of most rotor lifted aircraft in hover-
ing, and unsatisfactory dynamics of aircraft in hovering are more or
less inevitable unless $m_u$ can be maintained at a small value.   $x_u$
provides a small stabilizing effect on the oscillation, and a small
range of $m_u$ for which the machine could be dynamically stable.   The
pitch damping ( $m_{\dot\theta}$ ) also acts to stabilize the motion and its influ-
ence is dependent upon  $x_u$ to some extent.

12

Note that although the velocity stability is stable in a static sense it is destabilizing in a dynamic sense.  •

Expanding (4) along the $\alpha$ column, the three degree of freedom characteristic equation can be written as:

$$-X_\alpha \begin{vmatrix} z_u & \lambda \\ m_u & (m_{\dot\theta}-\lambda)\lambda \end{vmatrix} + (z_\alpha - \lambda) \begin{vmatrix} x_u-\lambda & -g \\ m_u & (m_{\dot\theta}-\lambda)\lambda \end{vmatrix}$$

$$- m_\alpha \begin{vmatrix} x_u-\lambda & -g \\ z_u & \lambda \end{vmatrix} = 0$$

In many cases $X_\alpha$ will be small and the characteristic equation, in literal terms becomes

$$(z_\alpha - \lambda) \left[ \text{hovering minor} \right] - m_\alpha \left[ \text{phugoid minor} \right] \cong 0 \quad (6)$$

Thus as $m_\alpha \rightarrow 0$ , the dynamics of the machine tend towards this minor.

2.  Classical Phugoid Minor:

$$\begin{vmatrix} (x_u-\lambda) & -g \\ z_u & \lambda \end{vmatrix} = 0$$

This minor describes the motion of the aircraft when the angle of attack influence is small in the horizontal and vertical force e-quations, and is usually a good approximation to one mode of the

13

dynamics of an aircraft or helicopter possessing a large amount of angle of attack stability ( $m_\alpha$ ) as can be seen from equation (6). The nature of the characteristic roots of this minor depend primarily on $Z_u$ since $X_u$ being a function of the drag of the machine, will be of the same sign and of similar magnitude on most aircraft. For a subsonic airplane $Z_u$ is always negative. However, in the helicopter, $Z_u$ may be either positive or negative, depending upon the flight condition. (References 3, 4, and 5). If $Z_u$ is negative, i.e., a lift increase with an increase in speed, the roots will consist of two convergences tending to a stable oscillation as $Z_u$ increases. If $Z_u$ is positive, the roots consist of a convergence and a divergence. The former is generally the situation at low trim velocities and the latter at high trim velocities for a helicopter. The locus of roots is therefore:
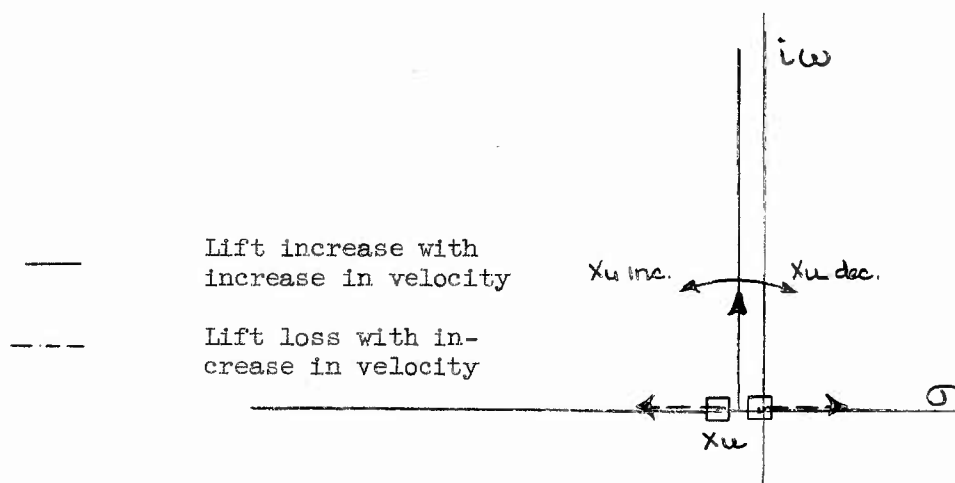


Figure 2: Classical Phugoid Dynamics

14

Variation of $X_u$ from its original value moves the roots on the arc of a circle centered at the origin, when the initial roots are oscillatory.

This case is fundamentally different from the hovering minor although in both cases the angle of attack influence was assumed to be small. This is due to the fact that different equations are involved. Thus even though the angle of attack variation is small, one must be careful to choose the equations in which the forces or moments produced by the angle of attack change are small compared to the other terms in the equation. The selection of the proper equations for an approximation follow directly from determination of mode ratios.

3. Classical Short Period Minor:

$$\begin{vmatrix} Z_\alpha - \lambda & 1 \\ m_\alpha & m_{\dot\theta} - \lambda \end{vmatrix} = 0$$

This minor describes the short time dynamics of an aircraft, that is, the motion prior to the time that the velocity change has increased to a sufficient magnitude to influence the dynamics. In general, if the frequency of this motion is high and well damped it will represent a good approximation to one pair of roots of the three degree of freedom characteristic equation, since the oscillation would ensue before a significant velocity change occurs. If the frequency is low this minor may not be a good approximation to roots of the characteristic equation but may still approximate the short time dynamics of the aircraft.

15

As mentioned $m_{\dot\theta}$ will in general be considerably larger than $x_u$ for a majority of aircraft and helicopters. The lift curve slope of the aircraft ( $z_\alpha$ ) will be of a similar magnitude as $m_{\dot\theta}$ representing in an uncoupled situation, a rapid convergence in angle of attack. Thus the roots of this minor will depend, to a large extent, upon the magnitude and sign of $m_\alpha$ as shown on the following root locus. Also indicated are the influences of variation of $z_\alpha$ and $m_{\dot\theta}$ in an oscillatory case. An $m_{\dot\theta}$ change moves the roots on the arc of a circle centered at $z_\alpha$ and vice versa.
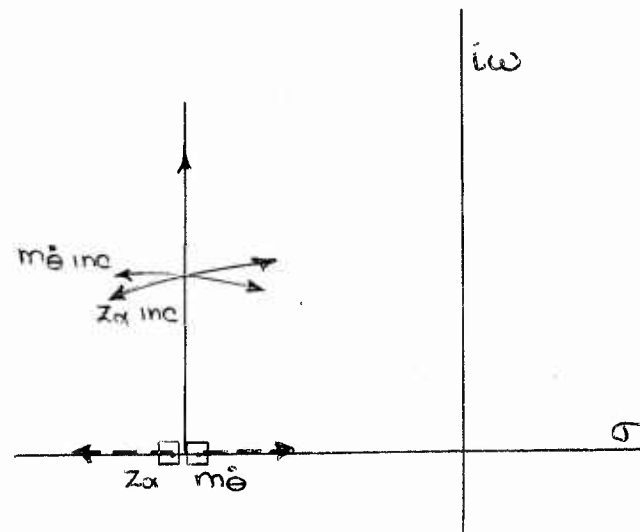


---- angle of attack instability

_____ angle of attack stability

Figure 3: Classical Short Period Dynamics

For large values of angle of attack stability the roots will be the typical short period heavily damped motion of the subsonic airplane providing $Z_\alpha$ and $m_{\dot\theta}$ are of sufficient magnitude. If there is angle of attack instability, then the roots will be two convergences, one tending towards a divergence as the instability increases.

Now the over-all dynamic characteristics as obtained from the 3 x 3 determinant will be investigated. Certain basic relationships between the derivatives exist that are utilized to make the root locus diagrams quite general with regard to the influence of other derivatives. One would expect, in particular, that for similar types of aircraft, e.g., single rotor helicopters, the force equations would be similar since the derivatives would largely result from performance considerations. There may or may not be similarities in the moment equation depending upon the type of aircraft and the configuration under consideration, and upon the degree of control that can be exerted in the design. As previously mentioned, the stability derivatives along the major diagonal of the determinant (4), from physical considerations can be expected to bear a general relationship to one another. These terms determine the dynamics when the degrees of freedom are not coupled. Thus in this situation, it would be expected that all aircraft would possess similar dynamics. If the velocity stability ($m_u$) and the angle of attack stability ($m_\alpha$) are equal to zero the pitching mode is uncoupled from the angle of attack and velocity modes.

17

When $m_u = m_\alpha = 0$ , (4) becomes

$$\begin{vmatrix} (x_u - \lambda) & x_\alpha & -g \\ z_u & (z_\alpha - \lambda) & \lambda \\ & & (m_{\dot\theta} - \lambda)\lambda \end{vmatrix} = 0$$

and the characteristic equation is:

$$\lambda (m_{\dot\theta} - \lambda) \left\{ (z_\alpha - \lambda)(x_u - \lambda) - x_\alpha z_u \right\} = 0 \qquad (7)$$

The angle of attack and the velocity are coupled due to $z_u$ and

$x_\alpha$ . It will be found that as long as $x_u$ and $z_\alpha$ are well

separated, this coupling between angle of attack and velocity is

usually weak and can be neglected. This is generally true of both

the airplane and the helicopter, and is determined by the condition

that the roots of $(x_u - \lambda)(z_\alpha - \lambda) - x_\alpha z_u = 0$ are

approximately $x_u$ and $z_\alpha$ . In this case, the characteristic

equation will be $\lambda (m_{\dot\theta} - \lambda)(z_\alpha - \lambda)(x_u - \lambda) = 0$

Thus the dynamics of any airframe with no angle of attack sta-

bility and no velocity stability will be essentially uncoupled and

will consist of:

  a)  In pitching rate, a rapid convergence.

  b)  In angle of attack, a rapid convergence.

  c)  In velocity, a slow convergence.

This basic relationship is inherent in the classical approximations as

will be seen. There will, of course, be some interaction between the

equations due to the gravity and inertia terms. For example, the

weight component term in the horizontal force equation $(-W \Delta\theta)$ will

act as a forcing function in the evaluation of control response in the

uncoupled situation.

18

The dynamics of the helicopter or airplane will be largely controlled by the magnitude and sign of $m_u$ and $m_\alpha$ , and the root location for the uncoupled situation is typical of most aircraft, appearing on the complex plane as follows from equation (7). The zero root arises from the fact that the moment equation is uncoupled and has no dependence upon pitch angle.
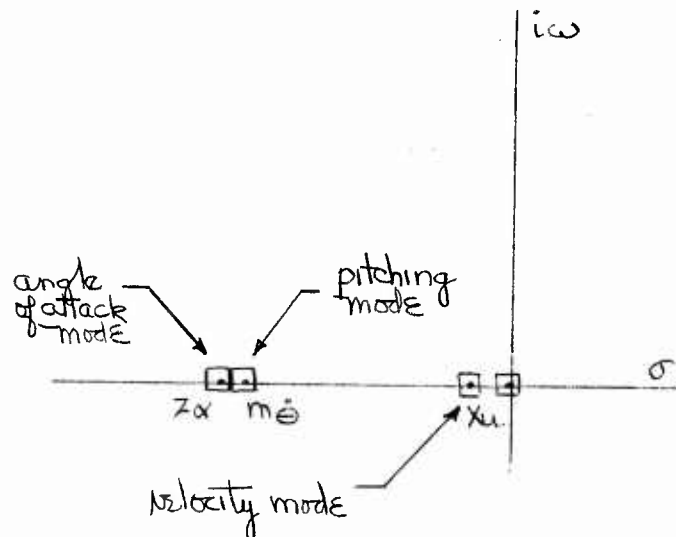


Figure 4: The dynamics of an aircraft with no velocity stability and no angle of attack stability $(m_\alpha = m_u = 0)$

Generally these time constants will be well separated as shown. ( $z_\alpha$ may be greater or less than $m_{\dot\theta}$ ). However, the typical helicopter usually has smaller values of $z_\alpha$ and $m_{\ddot\theta}$ than the airplane. For the helicopter through the level flight speed range this configuration will remain approximately the same. $|x_u|$ will increase with speed while $m_{\dot\theta}$ and $z_\alpha$ will remain approximately constant.

19

Now consider in what manner the variable phase and magnitude relationships are dependent upon the characteristic roots. It is necessary to use only two of the three equations of motion. Since the character of the force equations is typical and the static stability derivatives will be varied, relationships between the variables for any location on the complex plane are obtained from the force equations. The force equations are:

$$(X_u - \lambda) \frac{u_1}{\Theta_1} + X_\alpha \frac{\alpha_1}{\Theta_1} = g$$

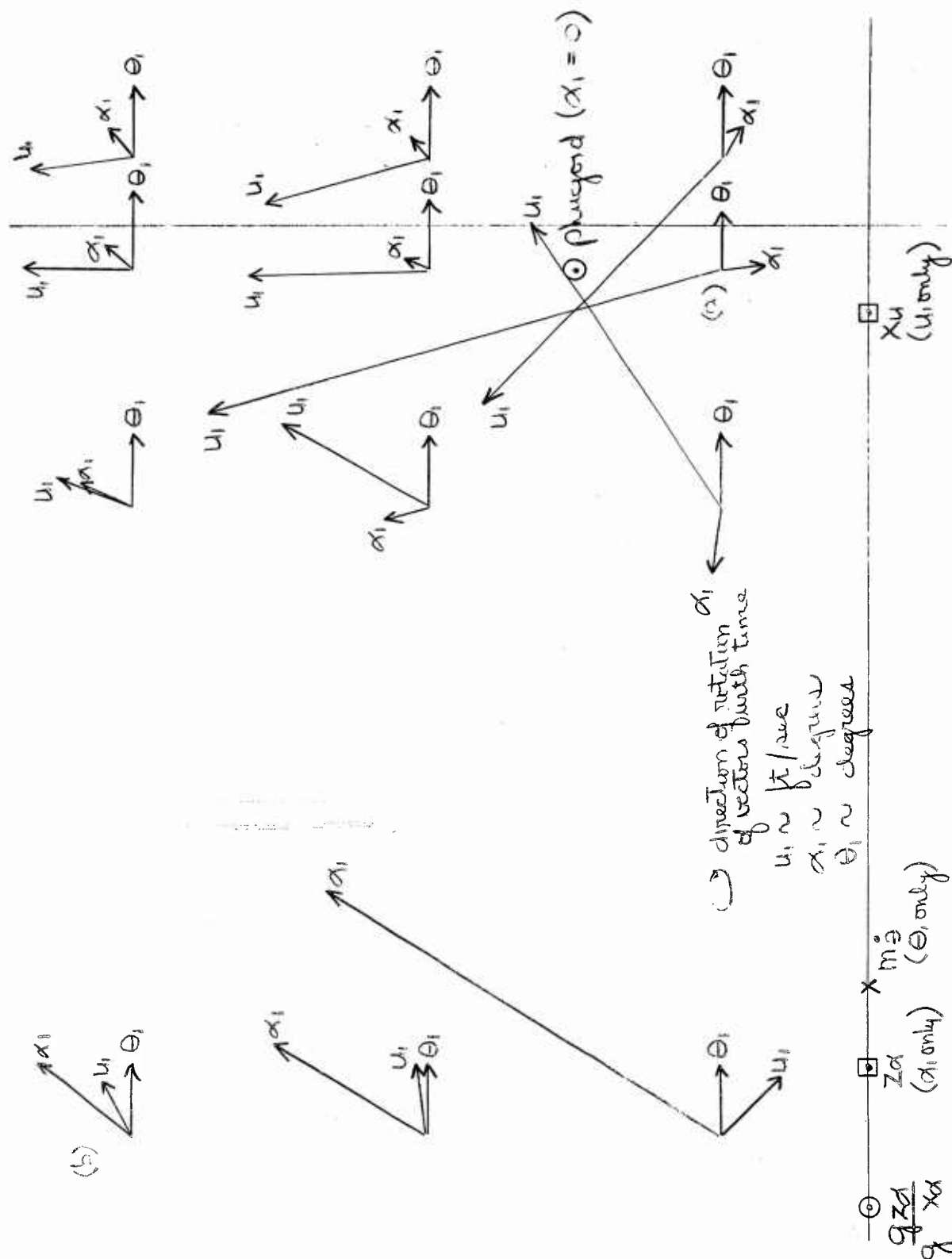$$Z_u \frac{u_1}{\Theta_1} + (Z_\alpha - \lambda) \frac{\alpha_1}{\Theta_1} = -\lambda \tag{8a}$$

and

$$\frac{u_1}{\Theta_1} = \frac{\lambda(X_\alpha - g) + g Z_\alpha}{(X_u - \lambda)(Z_\alpha - \lambda) - X_\alpha Z_u}$$

$$\frac{\alpha_1}{\Theta_1} = \frac{-g Z_u - \lambda(X_u - \lambda)}{(X_u - \lambda)(Z_\alpha - \lambda) - X_\alpha Z_u} \tag{8b}$$

Thus each value of $\lambda$ determines a value of $\frac{u_1}{\Theta_1}$ and $\frac{\alpha_1}{\Theta_1}$. These quantities can be considered as vectors on the complex plane and computed from this viewpoint. For the approximations $\frac{X_\alpha}{g} << 1$ and that the roots of $(X_u - \lambda)(Z_\alpha - \lambda) - X_\alpha Z_u = 0$ are approximately $X_u$ and $Z_\alpha$, the velocity to pitch angle relationship reduces to $\frac{u_1}{\Theta_1} = \frac{g}{(X_\alpha - \lambda)}$ and for any values of $\lambda$ large compared to $X_u$ this simplifies further to $\frac{u_1}{\Theta_1} = -\frac{g}{\lambda}$. This approximation is easily interpreted physically. It represents the fact that the major terms in the horizontal force equation are the horizontal force produced by the weight component along

20

the $\chi$ axis and the acceleration resulting therefrom. In this case, the horizontal acceleration, and the fuselage pitch angle will always be approximately out of phase. As the modes become faster and faster, i.e., as $\lambda$ increases, the velocity content of the mode becomes smaller and smaller.

The numerator of $\frac{\alpha_1}{\Theta_1}$ is the classical phugoid, and therefore as $\lambda$ approaches the classical phugoid $\frac{\alpha_1}{\Theta_1} \rightarrow 0$ i.e., there will be no angle of attack variation in that mode. When $Z_\alpha > X_u$ and the classical phugoid is oscillatory, the mode ratios on the complex plane are shown in Figure 5. For any given roots, the variable relationships are fixed. The magnitude and phase of a derivative is determined by changing the magnitude by $\sqrt{\omega^2 + \sigma^2}$ and advancing the vector counter-clockwise by the angle $\tan^{-1} \frac{\omega}{\sigma}$ . An estimate of the significance of the terms in various areas of the complex plane can be made by comparing the product of derivative of interest and the magnitude of the variable, e.g., $m_\alpha \frac{\alpha_1}{\Theta_1}$ to $m_u \frac{u_1}{\Theta_1}$ . Also the phase relationships are instructive as to similar effects from different derivatives. For example at (a) where the angle of attack and the velocity are $180^\circ$ out of phase it would be expected that a change in velocity stability (+) or a change in angle of attack stability (-) would have a similar influence on the dynamics. At (b) where the velocity and angle of attack are approximately in phase, their influence would be opposite. This is verified by the root loci later. Now the variable content as dependent upon the frequency and the damping of the mode can be seen. The lightly damped mode involves primarily velocity and pitch angle perturbations, angle of attack variations are of the

21

(c) direction of rotation $\alpha_1$
   of vectors with time

$u_1 \sim ft/sec$
$\alpha_1 \sim degrees$
$\theta_1 \sim degrees$

(b)

Figure 5: Mode Ratios

22

order of 1/2 the pitch angle variations, generally enough such that
the influence of angle of attack cannot be neglected except when the
static stability with angle of attack is large and the classical
phugoid roots are approached. The heavily damped motion consists
primarily of angle of attack change and pitch angle change and the
velocity change is essentially unimportant. Note that the associ-
ation of these mode characteristics with the roots is due only to the
force equation characteristics. The variable content of convergences
and divergences depend upon their location with respect to the un-
coupled dynamics, ( $X_u, Z_\alpha, m_{\dot\theta}$ ), e.g.. slow convergences will
have a significant velocity content, while fast convergences will not.

This property of the equations can be used in more complicated
problems to determine the composition of each mode.

Now the manner in which the dynamics vary with the velocity
stability ( $m_u$ ) and the angle of attack stability ( $m_\alpha$ ) will be
investigated. These two important terms in the equations of motion
vary considerably on helicopters and aircraft and may, to some extent,
be controlled in the design. Four situations are considered for angle
of attack stability: the influence of angle of attack stability and
instability when $Z_u$ is negative (the classical phugoid is oscillatory),
and when $Z_u$ is positive (the classical phugoid is a convergence and
a divergence); one for velocity stability: the influence of velocity
stability when $\dfrac{X_\alpha}{g}$ is not negligible compared to 1. The effect of
velocity stability when $\dfrac{X_\alpha}{g} << 1$ is indicated by the hovering
minor as the horizontal force and moment equations are not coupled to
the vertical force equation and ( $\lambda - Z_\alpha$ ) is a factor of the

23

characteristic equation as indicated by equation (7). These cases in
clude the consideration of the subsonic airplane (i.e., negligible ve-
locity stability) and the single rotor helicopter.

The characteristic equation for the following root loci is ob-
tained by expanding (4) along the moment row:

$$\lambda(m_{\dot{\theta}} - \lambda)\left\{(x_u - \lambda)(z_\alpha - \lambda) - x_\alpha z_u\right\}$$

$$-m_\alpha\left\{\lambda(x_u - \lambda) + g z_u\right\} - m_u g\left\{\lambda(1 - \frac{x_\alpha}{g}) - z_\alpha\right\} = 0$$

Assuming the term $x_\alpha z_u$ negligible and $x_\alpha \ll g$ as discussed
previously the characteristic equation becomes:

$$\lambda(m_{\dot{\theta}} - \lambda)(x_u - \lambda)(z_\alpha - \lambda) - m_\alpha\left\{(x_u - \lambda)\lambda + g z_u\right\}$$

$$-m_u g\left\{\lambda - z_\alpha\right\} = 0$$

### Dynamics as a function of angle of attack stability

Case 1:   Phugoid oscillatory; angle of attack stability

The locus of roots of the characteristic equation for all values
of $m_\alpha$ less than 0 will present the range of dynamics of the subsonic
airplane. The zeros of $m_\alpha$ are of the classical phugoid roots, and
the root locus is obtained from the characteristic equation

$$\lambda(m_{\dot{\theta}} - \lambda)(x_u - \lambda)(z_\alpha - \lambda) - m_\alpha\left\{\lambda(x_u - \lambda) + g z_u\right\} = 0$$
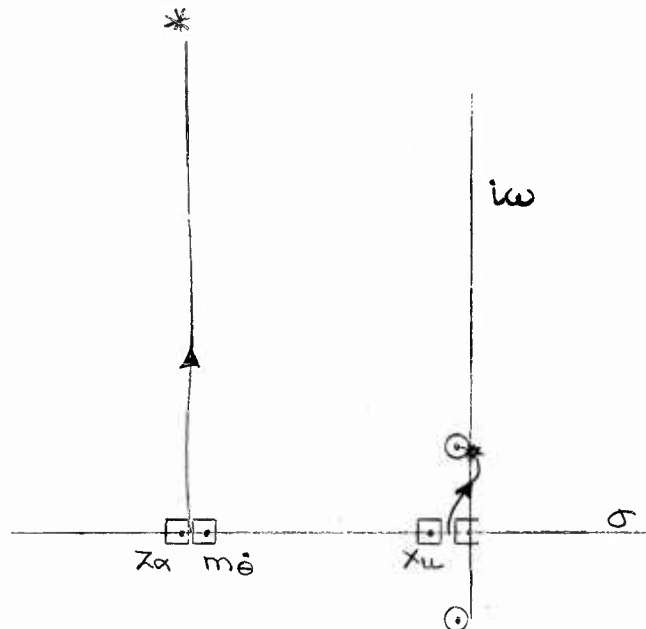
24

Figure 6a: The dynamics of an airframe as a function of angle
of attack stability. Phugoid oscillatory. $(m_\alpha < 0)$

This locus presents the characteristic roots of the longitudinal motion
of the subsonic airplane. Two oscillations are produced for any ap-
preciable stable value of $m_\alpha$ , one arising from the coupling of the
pitching rate mode and the angle of attack mode, the other arises
through coupling of the zero root and the velocity mode. These two
modes will be referred to as the heavily damped mode and the lightly
damped mode to distinguish them from the classical airplane modes. As
$m_\alpha$ is increased the classical picture is approached of a lightly
damped long period mode, (the limiting case is the classical phugoid
motion) and a heavily damped short period motion. Therefore, the
classical approximations depend upon a significant amount of angle of
attack stability and the relationship originally assumed that
$|m_{\dot{\theta}}|, |z_\alpha| \gg |x_u|$ . When these quantities are not well separated the

classical approximations will become less successful, and the tendency towards instability will increase in the light damped branch. The classical approximations become exact as $m_\alpha \rightarrow -\infty$. The actual damping of the phugoid is less than the classical phugoid approximation of $\frac{x_u}{2}$ due to coupling between the two modes and is difficult to approximate in a simple fashion. By examination of Routh's Discriminant, a criterion for the occurrence of instability can be obtained. If $\left| \frac{m_{\dot\theta} + z_\alpha}{m_{\dot\theta}\, z_\alpha} \right| > \left| \frac{z_u\, g}{4\, x_u} \right|$ then there will be a range of angle of attack stability in which the lightly damped oscillation is dynamically unstable.

Case 2: Phugoid oscillatory; angle of attack instability

The locus of roots is obtained from the previous characteristic equation by changing the angle condition.
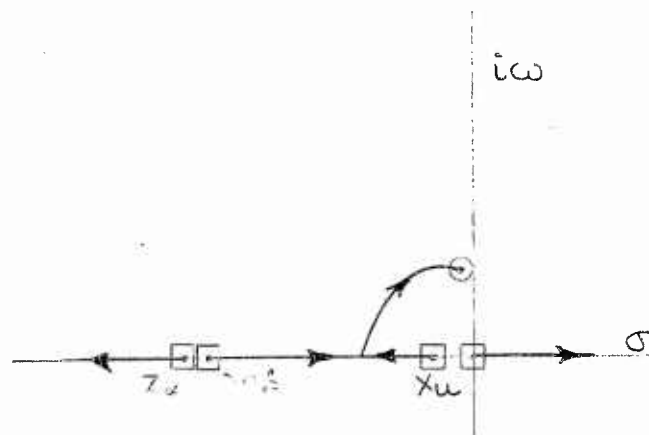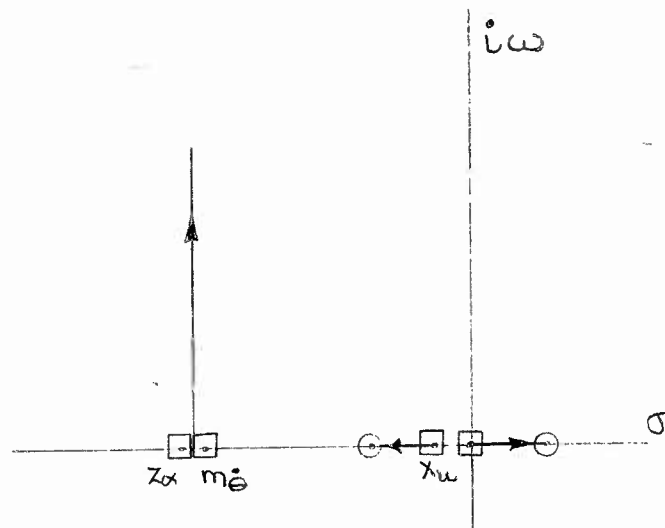


Figure 6b: Dynamics of an airframe as a function of angle of attack instability. Phugoid oscillatory. $(m_\alpha > 0)$

26

Here we obtain a heavily damped oscillatory mode, referred to as the "third mode" in (Reference 6) and a convergence and a divergence. The divergence is a result of the static instability of the airplane. It is interesting to note that, in this case, a large value of angle of attack instability also tends towards the classical phugoid mode.

Case 3:  Phugoid non oscillatory;  Angle of attack stability

The locus of roots is again obtained from the previous characteristic equation with new zeros due to the difference in $Z_u$ .
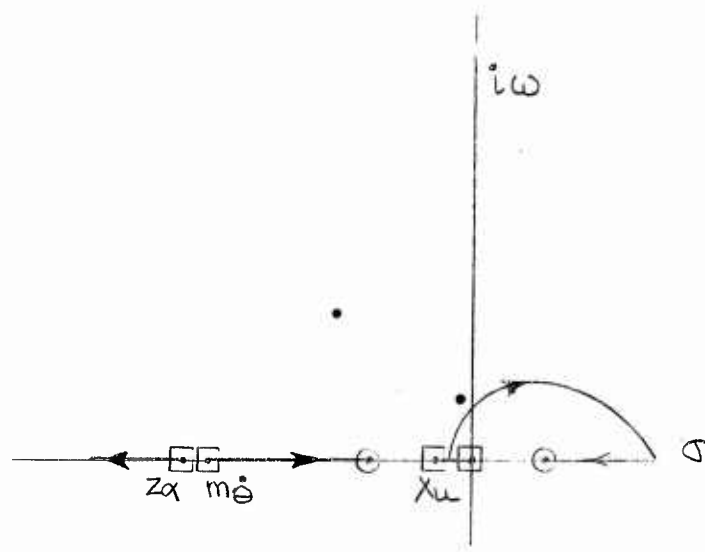


Figure 6c:  Dynamics of an airframe as a function of angle of attack stability.  Phugoid non oscillatory.  $(m_\alpha < 0)$

Here there is essentially no change in the heavily damped branch.
However, the lightly damped branch is quite different. The aircraft
is statically unstable , i.e., the coefficient of $\lambda^{o}$ in the charac-
teristic equation has changed sign due to $Z_{u}$ . Increasing the
angle of attack stability increases the rate of divergence of the
instability. It is interesting to note, however, that for reasonable
values of $m_{\alpha}$ the classical approximations still apply.

Case 4:    Phugoid non oscillatory; angle of attack instability

This root locus is similar to case 3, except that the angle con-
dition is changed.



Figure 6d:   Dynamics of an airframe as a function of angle of
attack instability. Phugoid non oscillatory.
$(m_{\alpha} > 0)$

28

Here, as in Case 2, the heavily damped branch in non-oscillatory, while the lightly damped branch is oscillatory, and surprisingly, may even be stable for small values of $m_\alpha$. For large values of angle of attack instability a severe oscillatory instability occurs, tending towards a rapid divergence.

### Dynamics as a function of Velocity Stability.

This situation differs from the hovering case as a result of $\frac{x_\alpha}{q}$ being significant (If $x_\alpha = m_\alpha = 0$ then angle of attack variation appears only in the vertical force equation. The oscillatory mode is described by the other two equations). This root locus is obtained from the characteristic equation in the form

$$\lambda ( m_{\dot\theta} - \lambda)(x_u - \lambda)(z_\alpha - \lambda) + m_u \left\{ \lambda(-g + x_\alpha) + g z_\alpha \right\} = 0$$
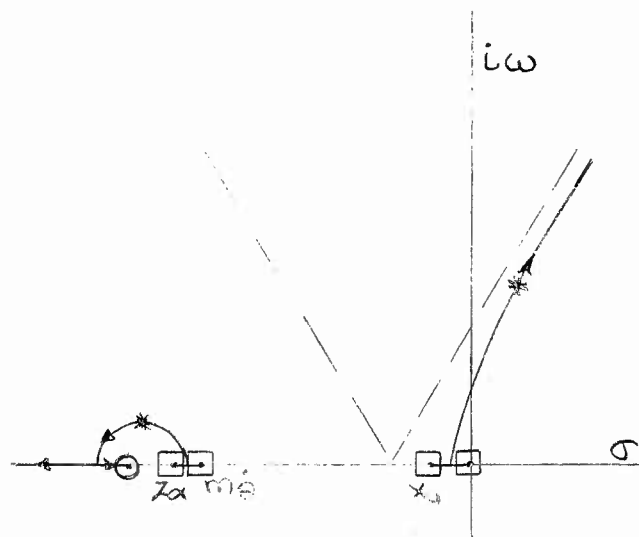
Figure 7: The dynamics of an airframe as a function of Velocity Stability. $m_u > 0$

29

The lightly damped or unstable branch is quite similar to the hovering situation, while the heavily damped branch may either consist of a heavily damped long period oscillation as shown or two convergences, depending upon the sign and magnitude of $X_\alpha$ . Thus when $m_\alpha \to 0$ the "hovering" dynamics represent a good approximation to the dynamics of the helicopter in forward flight. The actual dynamics differ from the "hovering" dynamics primarily due to angle of attack stability. Also note that in the situation when $\frac{X_\alpha}{g} << 1$ , $m_\alpha = 0$ , $(\lambda - Z_\alpha)$ is a factor of the characteristic equation.

Thus the manner in which the dynamics of the airframe vary with angle of attack stability and instability and with varying degrees of velocity stability has been shown.

Comparisons of Figure 6a with Figure 7 demonstrates the difference between the longitudinal dynamics of an aircraft when the static stability of the aircraft is due to angle of attack stability, and when it is due to velocity stability. There is one region of similarity when the velocity stability is very small, its variations affect the lightly damped mode in a similar fashion to the variations of angle of attack stability. This region is restricted approximately to the period of dynamics where classical phugoid is oscillatory and the period of the lightly damped motion is longer than the classical phugoid period. The roots indicated on these two Figures by an asterisk, are a comparison of dynamics when the static margin of the airplane (Reference 7) is the same, but arises from these two different sources.

Now the influence of combinations of these two derivatives will
be considered. In Figure 8a the influence of angle of attack sta-
bility and instability for various values of velocity stability is
indicated, and in Figure 8b, the influence of velocity stability at
various values of angle of attack stability is presented. Only the
situation where the classical phugoid is oscillatory is considered
here.

Discussion will be restricted to oscillatory characteristics.
Corresponding roots on the two branches can be estimated from the
fact that the sum of the damping of the two modes is a constant when
only the static stability derivatives are varied.

The poles for the following root loci are obtained from previous
diagrams and the zeros are the same as before. For example, the poles
for Figure 8a are obtained from Figure 7 for various values of $m_u$,
and the zeros are the same as in Figure 6a.
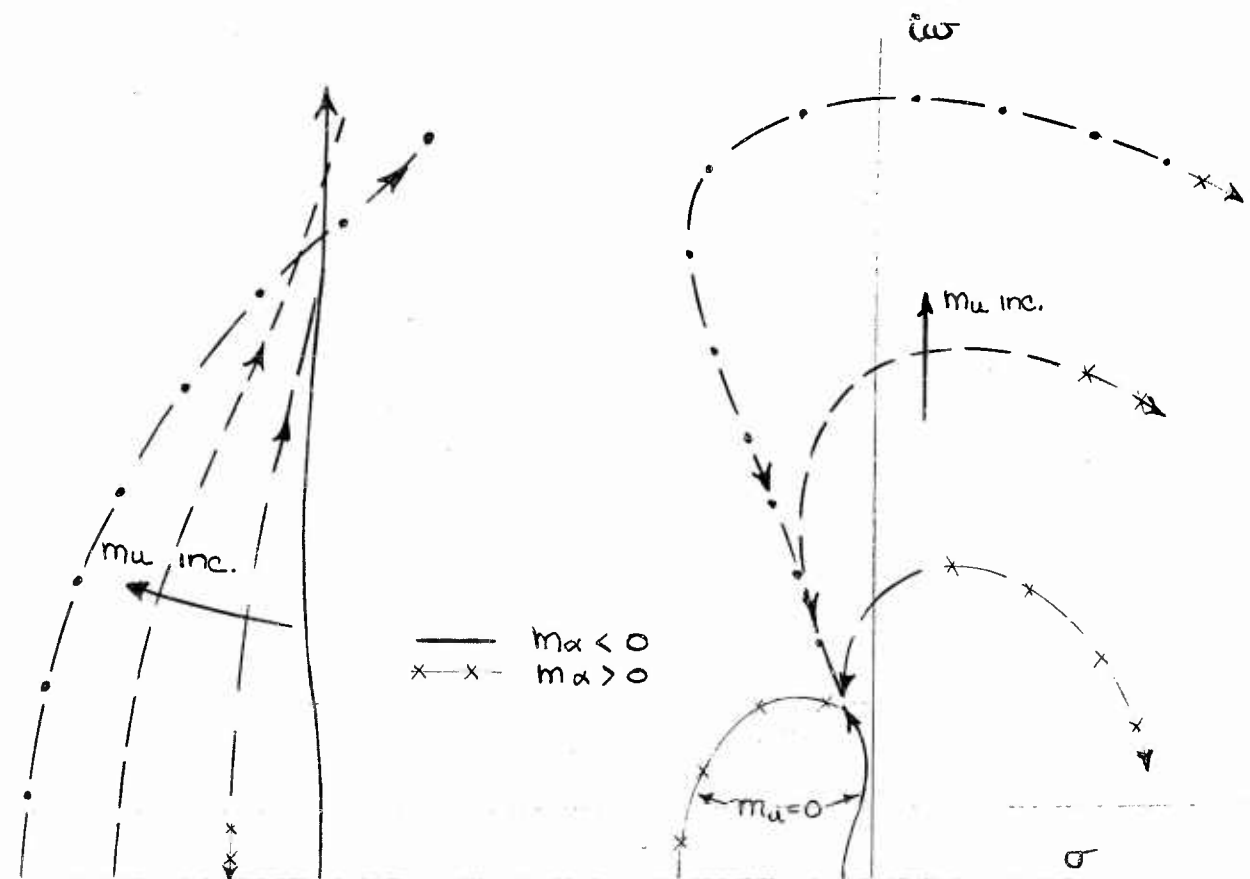
Figure 8a:   The dynamics of an airplane as a function of angle of
             attack stability for different velocity stability.
             (Phugoid oscillatory)

There are two regions to consider with regard to the lightly

damped mode:  One previously mentioned, when the frequency of the

motion is less than the classical phugoid, and the other when the

frequency is greater than the classical phugoid frequency.  In the

former region, the motion is of long period and slightly stable.

Both $m_\alpha$ and $m_u$ produce similar influences upon the dynamics,

reducing the period and the damping.  As the velocity stability in-

creases, the period becomes less than the classical phugoid, and the

32

influence of angle of attack stability becomes more and more bene-
ficial, decreasing the instability and lengthening the period, always
tending towards the classical phugoid. The influence of angle of
attack instability becomes more and more severe, strongly increasing
the dynamic instability and lengthening the period somewhat. In-
creasing the velocity stability is rarely beneficial, except in
regions where there are large amounts of angle of attack stability.
Even in this condition it is hardly desirable since it primarily re-
duces the period of a lightly damped oscillation, changing the damping
very little.
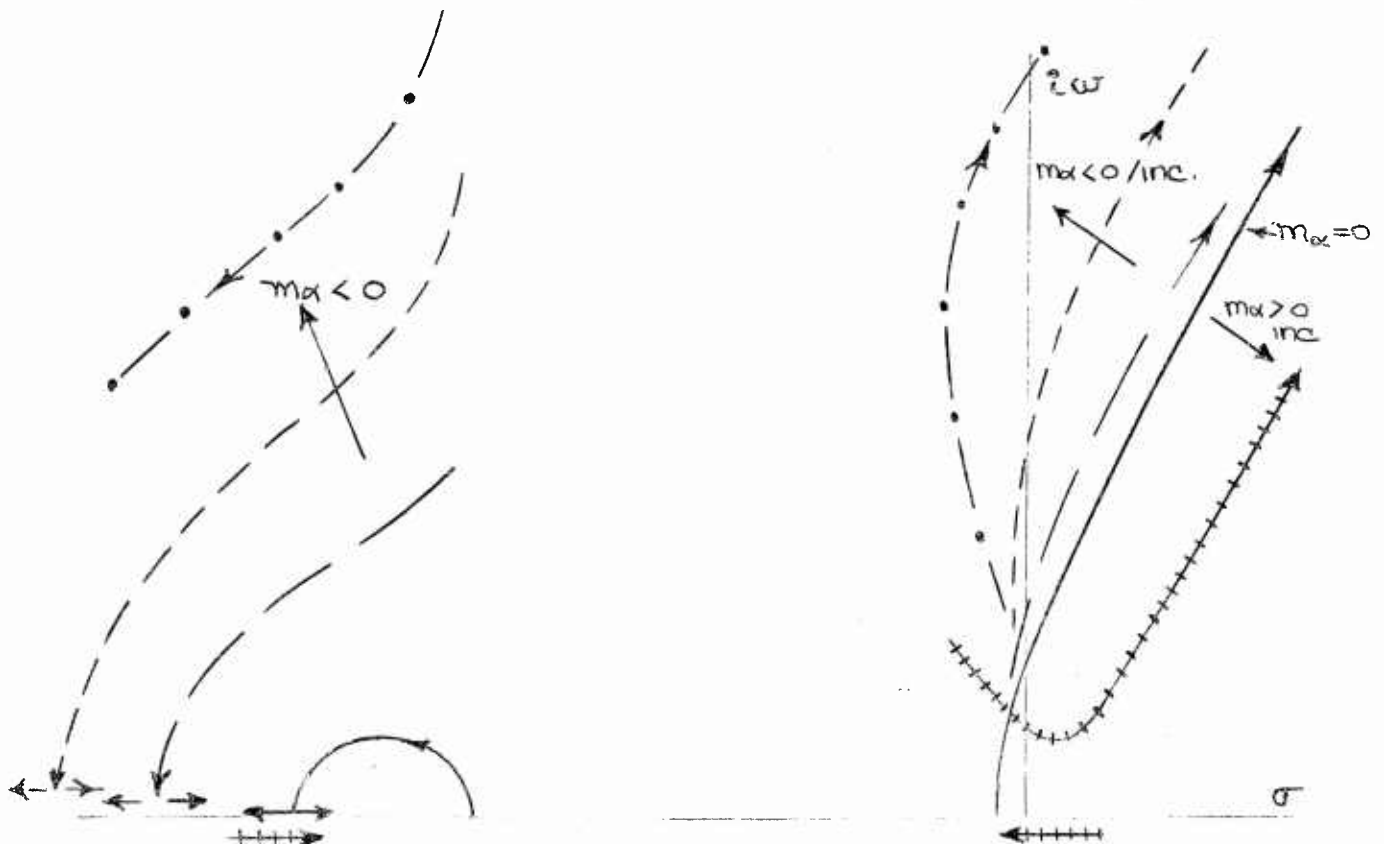


Figure 8b: The dynamics of an airplane as a function of velocity
stability for various values of angle of attack
stability and instability.

33

If the phugoid is not oscillatory the situation will be similar. The significant difference occurs due to the fact that increasing the angle of attack stability will eventually cause a divergence to occur when the coefficient of $\lambda^c$ changes sign. Instability occurs when $|m_\alpha z_u| > |m_u z_\alpha|$ the influence of both derivatives is similar to the previous case except that there is no region of similarity in the lightly damped mode. Increases in $m_u$ result in oscillatory instability and reduction in period. Angle of attack stability stabilizes the motion and always increases the period, and a large degree may cause static instability of the airplane.

Thus desirable characteristics of the long period motion are obtained by maintaining $m_u$ as small as possible, and obtaining a large amount of angle of attack stability. However, in the situation where $z_u$ is such that there is a lift loss with increase in speed maintaining $|m_u z_\alpha| > |m_\alpha z_u|$. In this latter situation it may be desirable to increase the velocity stability to prevent this dynamic instability. This is perhaps the only situation in which an increase in velocity stability is desirable.

Now the heavily damped mode will be considered. If $m_{\ddot\theta}$ and $z_\alpha$ are not large then a large degree of angle of attack stability may make the damping ratio of the heavily damped mode small enough to be undesirable. If the criterion of Reference 8 is satisfied with regard to $m_{\ddot\theta}$, then this should not be a problem. Increasing angle of attack stability always raises the frequency of the heavily damped mode. An increase in velocity stability lowers the frequency and generally increases the damping of the heavily damped mode.

34

From these considerations it can be seen that the comments of Reference 9 with regard to the modes of the HO-3S helicopter would be generally true for any aircraft with a significant amount of velocity stability and insufficient angle of attack stability.

The influence of other stability derivatives may be determined from root locus techniques. However, for an over-all view we can see the primary influences from the previous root loci. $X_u$ provides damping of the lightly damped mode. ( $\frac{X_u}{2}$ in most cases is the maximum amount of damping). It has little influence on the heavily damped mode. $m_{\dot\theta}$ and $Z_\alpha$ govern the damping of the heavily damped mode and influence the lightly damped mode. Increase in pitch damping ( $m_{\dot\theta}$ ) is always helpful with regard to the lightly damped mode, increasing the period and damping. However, extremely large values will be required to stabilize this mode unless $X_u$ is large. The influence of $Z_\alpha$ on the lightly damped mode depends upon the values of $m_u$ and $m_\alpha$ since the minor of $Z_\alpha$ is the "hovering" minor. If angle of attack stability is present, the damping of the lightly damped mode is better than the hovering case and thus an increase in $Z_\alpha$ will be destabilizing. For angle of attack instability, an increase in $Z_\alpha$ will be stabilizing. The importance of this effect depends upon the size of $m_\alpha$ .

An estimate of the importance of derivatives such as $m_{\dot\alpha}$ may be determined by noting the root of $m_{\dot\alpha} \lambda + m_\alpha = 0$ . If this root $\lambda = - \frac{m_\alpha}{m_{\dot\alpha}}$ is of the order of the roots under consideration on the root locus, then the influence of $m_{\dot\alpha}$ should be taken into account. If it is much larger than the magnitude of the roots

under consideration, then $m_{\ddot{\alpha}}$ may be neglected. It is on this basis that terms such as $x_{\ddot{\Theta}}$ can be neglected.

The mode ratios for all the foregoing root locus diagrams are indicated on Figure 5.

It has been shown that the character of longitudinal motion of an aircraft is largely determined by the static stability derivatives ( $m_u$ and $m_{\alpha}$ ). The characteristic roots normally fall into two groups:

1. A well damped oscillation or two comparatively fast convergences. Whether or not this mode is oscillatory depends primarily upon angle of attack stability. The damping of this mode is governed by the lift curve slope of the aircraft ( $Z_{\alpha}$ ) and the pitch damping ( $m_{\ddot{\Theta}}$ )

2. A lightly damped or unstable oscillation, or a slow convergence and divergence. The character of this mode depends to a large extent on both $m_u$ and $m_{\alpha}$. The maximum damping of this mode is dependent upon the drag.

If there are two oscillations, then except in the case where $m_u$ is very small, and the influence of $m_u$ and $m_{\alpha}$ on the lightly damped mode is similar, their effect on the frequency of these two oscillations is opposite. An increase in angle of attack stability raises the frequency of the heavily damped mode, and lowers the frequency of the lightly damped mode. Increase in velocity stability raises the frequency of the lightly damped mode and lowers the frequency of the heavily damped mode. It appears then, that unless a large amount of angle of attack stability is present, if two oscillations are present, they will tend to be of similar period.

36

Now the motion of the single rotor helicopter will be examined.
The single rotor helicopter will have smaller values of $M\overset{\circ}{\theta}$ and
$Z_\alpha$ than the airplane. However, from the foregoing root loci,
this is not a fundamental difference. The fundamental differences
lie in the static stability derivatives and the fact that at high
speeds there can be a lift loss with increase in speed. One view-
point, since the single rotor helicopter will usually possess a sig-
nificant amount of velocity stability, is to consider the hovering
motion described by the hovering minor as basic. (As a rough approxi-
mation, $M_u$ and $M\overset{\circ}{\theta}$ may be considered constant with forward
speed). The dynamics in this case will consist of an uncoupled con-
vergence in angle of attack, a convergence in pitch angle and angle
of attack, and an unstable oscillation involving all three variables.
Now the influence of angle of attack stability on this motion must
be considered. The typical variation of angle of attack stability as
a function of forward speed on a helicopter is as follows (References
4 and 9). Normally at very low speeds the angle of attack stability
will be negligible. At a somewhat higher speed, in the optimum con-
figuration it is possible to obtain some angle of attack stability
and, as speed increases, this will develop into a strong angle of
attack instability with no horizontal tail. A horizontal tail of
sufficient size, can reverse this trend and provide a large amount
of angle of attack stability (Reference 4). With no tail, at low
speeds there may be two oscillations present, usually of similar
frequency, since $M_\alpha$ is not large enough to make the heavily damped
mode of the high frequency typical of the airplane. The lightly

37

damped mode may be approximately neutrally stable. At high speeds, there will be two convergences, one considerably faster than the other, and an unstable oscillation. With the horizontal tail the situation can be altered to produce two oscillations at high speed, one well damped and of somewhat shorter period than the other. The lightly damped motion may even become two convergences, or a convergence and a divergence for sufficiently large $m_\alpha$ when a lift loss with speed is present.

It is difficult to determine a second order approximation to the lightly damped motion when there is a significant amount of velocity stability present. The approximation suggested in Reference 10 applies only when the period of the motion is very long or when there is a large degree of angle of attack stability and is misleading particularly with regard to the influence of velocity stability on the damping of the motion in the case of the typical helicopter. Regions of validity of various approximations can be rapidly estimated by inspection of the characteristic equation in factored form. For example the characteristic equation of (4) may be written as:

$$\lambda(\lambda - m_{\dot\theta})(\lambda - z_\alpha)(\lambda - x_u) + m_\alpha \left\{ \lambda(\lambda - x_u) + z_u\, g \right\} + m_u g \left\{ \lambda - z_\alpha \right\} = 0$$

and in the region of the heavily damped mode in Figure 8a

$$|\lambda| \gg |x_u| \quad \text{and} \quad |\lambda|^2 \gg |g\, z_u| \quad , \text{ therefore}$$

$$\lambda^2 (\lambda - m_{\dot\theta})(\lambda - z_\alpha) + m_\alpha \lambda^2 + m_u g (\lambda - z_\alpha) = 0$$

simplifying the equation to some degree, and indicating the influence

38

of $m_u$ on the heavily damped mode. Now if $|\lambda|^2$ is large and $|\lambda| \cong z_\alpha$ this reduces to the short period approximation

$$\lambda^2 \left[ (\lambda - m_{\dot\theta})(\lambda - z_\alpha) + m_\alpha \right] = 0$$

In this way the consistancy and range of application of various approximations for various roots can be quickly estimated and the important terms contributing to the mode can be seen.

Thus, in conclusion, a convenient viewpoint for the longitudinal dynamics has been presented which makes it possible to obtain a good physical basis from which to consider the dynamics of the airplane, and in particular to visualize in a general way the influence of the static stability derivatives on airplane motion.

1.) The longitudinal dynamics of an aircraft consist of a heavily damped oscillation (or two convergences) and a lightly damped or unstable oscillation in the usual case where $\left|\dfrac{M_{\dot{\theta}}}{I}\right|$, $\left|\dfrac{Z_{\alpha}}{mU_{0}}\right| >> \left|\dfrac{X_{u}}{m}\right|$ and the aircraft is statically stable. The validity of the classical short period and phugoid approximations depend upon the above relationship and upon angle of attack stability. The larger the angle of attack stability, and the greater the separation of $\dfrac{M_{\dot{\theta}}}{I}$ and $\dfrac{Z_{\alpha}}{mU_{0}}$ from $\dfrac{X_{u}}{m}$, the better the approximations.

2.) The presence of velocity stability tends to invalidate the classical approximations and influences, in particular, the lightly damped motion, decreasing the period and making the motion unstable.

3.) Both modes of motion occur in all three variables. The velocity and the pitch angle predominate in the lightly damped motion, and the angle of attack and pitch angle predominate in the heavily damped motion. As the classical approximations are approached, the velocity change becomes negligible in the heavily damped mode and the angle of attack change negligible in the lightly damped mode.

40

# REFERENCES

1. Evans, W.R.; Control System Dynamics, Chapter 7, McGraw-Hill, 1954.

2. Gessow, A. and Myers, G.C.; Aerodynamics of the Helicopter, Chapter 2, Macmillan, 1952.

3. Shapiro, J.; Principles of Helicopter Engineering, Page 56, McGraw-Hill, 1955.

4. Bramwell, A.R.S.; Longitudinal Stability and Control of the Single Rotor Helicopter, Aeronautical Research Council of Great Britain, R. & M. 3104, 1959.

5. Squire, H.B.; Fail, R.A.; Eyre, R.C.W.; Wind-Tunnel Tests on a 12-foot Diameter Helicopter Rotor; Aeronautical Research Council of Great Britain, R. & M. 2695, 1953.

6. Etkin, B.; Dynamics of Flight, John Wiley & Sons, 1959.

7. Gates, S.B. and Lyon, H.M.; A Continuation of Longitudinal Stability and Control Analysis, Aeronautical Research Council of Great Britain, R. & M. 2027, 1944.

8. Tapscott, R.J.; Criteria for Control and Response Characteristics of Helicopters and VTOL Aircraft in Hovering and Low Speed Flight. IAS Paper No. 60-51.

9. Schultz, E.R.; The Determination of Helicopter Longitudinal Stability Derivatives from Flight Test Data, Part III, WADC Technical Report 55-438, Part III, 1959.

10. Ashkenas, I.L.; McRuer, D.T.; Approximate Airframe Transfer Functions and Applications to Single Servo Control Systems, WADC Technical Report 58-82.

ALART PROGRAM
Technical Report
Distribution List

|  | ADDRESS | NO. OF COPIES |
|---|---|---|
| 1. | Chief of Transportation<br>Department of the Army<br>Washington 25, D.C.<br>ATTN: TCACR | 2 |
| 2. | Commander<br>Wright Air Development Division<br>Wright-Patterson Air Force Base, Ohio<br>ATTN: WCLJA | 2 |
| 3. | Commanding Officer<br>U.S. Army Transportation Research Command<br>Fort Eustis, Virginia<br>ATTN: Research Reference Center<br>ATTN: Aviation Directorate | 4<br>3 |
| 4. | U.S. Army Representative<br>HQ AFSC (SCR-LA)<br>Andrews Air Force Base<br>Washington 25, D.C. | 1 |
| 5. | Director<br>Air University Library<br>ATTN: AUL-8680<br>Maxwell Air Force Base, Alabama | 1 |
| 6. | Commanding Officer<br>David Taylor Model Basin<br>Aerodynamics Laboratory<br>Washington 7, D.C. | 1 |
| 7. | Chief<br>Bureau of Naval Weapons<br>Department of the Navy<br>Washington 25, D.C.<br>ATTN: Airframe Design Division<br>ATTN: Aircraft Division<br>ATTN: Research Division | 1<br>1<br>1 |
| 8. | Chief of Naval Research<br>Code 461<br>Washington 25, D.C.<br>ATTN: ALO | 1 |

| ADDRESS | NO. OF COPIES |
|---|---|

9.  Director of Defense Research and Development
    Room 3E - 1065, The Pentagon
    Washington 25, D.C.
    ATTN:  Technical Library                                    1

10. U.S. Army Standardization Group, U.K.
    Box 65, U.S. Navy 100
    FPO New York, New York                                      1

11. National Aeronautics and Space Administration
    1520 H Street, N.W.
    Washington 25, D.C.
    ATTN:  Bertram A. Mulcahy
           Director of Technical Inofrmation                    5

12. Librarian
    Langley Research Center
    National Aeronautics & Space Administration
    Langley Field, Virginia                                     1

13. Ames Research Center
    National Aeronautics and Space Agency
    Moffett Field, California
    ATTN:  Library                                              1

14. Armed Services Technical Information Agency
    Arlington Hall Station
    Arlington 12, Virginia                                      10

15. Office of Chief of Research and Development
    Department of the Army
    Washington 25, D.C.
    ATTN:  Mobility Division                                    1

16. Senior Standardization Representative
    U.S. Army Standardization Group, Canada
    c/o Director of Weapons and Development
    Army Headquarters
    Ottawa, Canada                                              1

17. Canadian Liaison Officer
    U.S. Army Transportation School
    Fort Eustis, Virginia                                       3

ADDRESS                                               NO. OF COPIES

18.  British Joint Services Mission
     (Army Staff)
     DAQMG (Mov & Tn)
     1800 "K" Street, N.W.
     Washington 6, D.C.
     ATTN:  Lt. Col. R.J. Wade, R.E.                       3

19.  Office of Technical Services
     Acquisition Section
     Department of Commerce
     Washington 25, D.C.                                   2

20.  Librarian
     Institute of the Aeronautical Sciences
     2 East 64th Street
     New York 21, New York                                2

21.  U.S. Army Research and Development Liaison Group
     APO 79
     New York, New York
     ATTN:  Mr. Robert R. Piper                            1

AD _____ Accession No. _____

Princeton University Aero. Eng.
Dept., Princeton, N. J.

SOME BASIC CONSIDERATIONS REGARDING THE
LONGITUDINAL DYNAMICS OF AIRCRAFT AND
HELICOPTERS - Howard C. Curtiss, Jr.

Report No. 562, July 1961
41 pp.-illus.

Contract No. DA44-177-TC-524
Project No. 9-38-01-000, TK902
Unclassified Report

AD _____ Accession No. _____

Princeton University Aero. Eng.
Dept., Princeton N. J.

SOME BASIC CONSIDERATIONS REGARDING THE
LONGITUDINAL DYNAMICS OF AIRCRAFT AND
HELICOPTERS - Howard C. Curtiss, Jr.

Report No. 562, July 1961
41 pp.-illus.

Contract No. DA44-177-TC-524
Project No. 9-38-01-000, TK902
Unclassified Report

A discussion of the longitudinal motion of an airframe is presented. General relationships between the stability derivatives of the airplane and the single rotor helicopter are considered. It is shown that the basic character of the longitudinal motion is primarily determined by the angle of attack stability and the velocity stability. The variation in the modes of motion produced by these two stability derivatives is presented.

Consideration of the relationships between the flight variables in the modes of motion is included.

# UNCLASSIFIED